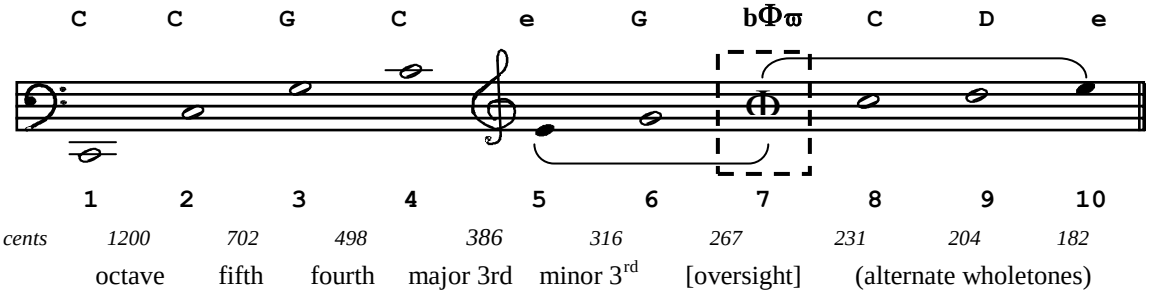
# The Harmonic Series As Universal Scientific Constant

Modern education emphasizes the *harmonic series* as establishing the *natural* foundation of quantification (numbered *partials* function as divisors of monochord string length), and so it is presented first as a common reference. Everything that follows concerns its colorful mythologizing in narrative allegory. Pitch notation now implies equal temperament tuning, 100 logarithmic *cents* per semitone, and so deviations are shown here in cents, for only the reference pitch (1200 cents to the octave) is accurately notated. The 7th partial--which plays a strange "oversight" role in ancient mythology--suffers most from this misrepresentation (it is perceptibly flat and is not adequately notated), but the 5th and 10th are somewhat less so. If the plucked string of a monochord (and a guitar serves conveniently) is touched very lightly at these fractional lengths then all other partials will be silenced; thus the string will prove that its fundamental tone is actually a composite that human ears accept as a unity. (Energy falls off rapidly in the higher partials, so the first few are easiest to isolate.) We are partners in the objects of our attention.

**Figure 1. The harmonic series as *natural* defining reference for pitch ratios.**

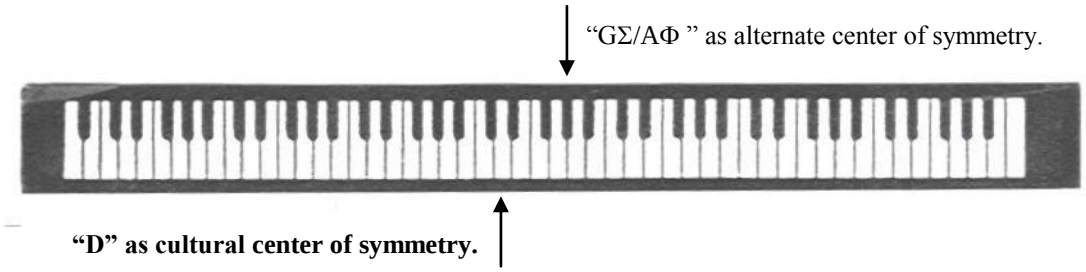



This deified Pythagorean "Ten-ness" inherited from the Marduk/Baal mythology of ancient Mesopotamia stops right here in identifying tuning ratios for two very good reasons, easily inferred, and beautifully illustrated by Ptolemy in the second century AD. 1) Once the music starts we normally cannot distinguish between wholetones of 8:9 and slightly smaller ones of 9:10, so that Ptolemy's example of a tetrachord (four consecutive tones) progressing 9:10, then 10:11, and finally 11:12 clearly spans a musical fourth of 3:4=9:12 and in a way that can beguile melodically, but it makes nonsense out of any discrimination between wholetones and semitones. But there is an even worse problem, 2) for the augmented fourth at 7:10 cannot be distinguished from the diminished fifth at 5:7, at least without reference to a third pitch, so that we can no longer even count sensibly. (Musicians habitually refer to both as "tritones," further confounding this issue.) Thus trouble lies ahead no matter how carefully we begin with perfect consonances accurately defined and labelled. This apparent "flaw" in Creation arises only because we love both perfect octaves and perfect fifths, fourths, and thirds, so that our *surfeit of perfections* conflicts with cyclic *Necessity* (deified by Plato in the 2:1 octave) "with whom not even the gods can contend." We are stuck with somewhat incompatible

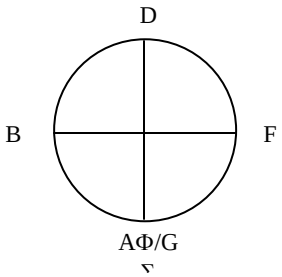
desires, a rather general condition of being human. In modern theory these partial numbers are ratios of frequency, reciprocals of string lengths.

What we hear as a "musical" tone turns out to be, on physical analysis, a "manifold" of partials with varying proportions of the total energy, so that hearing is always influenced by invisible and normally inaudible forces, a "magic" beyond our control except as, in performance, we become the magicians. Thus sound functions as the greatest clue to psychic "interiority" without ever fully disclosing its secrets.

A piano tuned in equal temperament (with 12 equal semitones to the octave) offers a convenient mapping of pitch classes through more than seven octaves. The niceties suggested by *cents* values are ignored for 87 consecutive semitones that preserve the illusion that octaves, fifths, and fourths are reasonably well-tuned, and we try to ignore what happens to thirds and sixths. (Players of all fretted instruments *fret* endlessly over the intrusion of mistuned thirds in particular.) The pattern of black and white digitals on a keyboard is a constant reminder that semitones occur at B:C and E:F after either two or three consecutive wholetones. The eye immediately locates two possible centers of symmetry in any twelve-tone "octave"—either on the white key "D" framed by a black pair or on the middle black key in any set of three and named either G-sharp or A-flat.




A persistent belief in the ancient Near East that the cosmos is a perfect inverse symmetry that unfolded originally from the middle makes it convenient to locate "Deity" on pitch class "D" and to "map" all tones on the local clock with D at the modern "zero hour" of 12:00 o'clock. The immediate result is to locate GΣ/AΦ directly opposite at 6:o'clock on a "moral" *plumb line* pointing to the square root of 2 that divides the octave *proportionally* into equal halves that we identify tonally as a *tritone* (meaning three wholetones). And dividing each tritone in the same way (now by the "fourth root of 2") locates our equal-tempered B and F on the horizontal "balance beam" dividing the cyclic octave into four equal tempered *minor thirds* (rising D:F, F:AΦ, as opposed to falling D:B and B:GΣ).

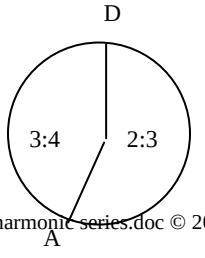


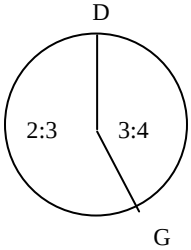

It is an historical coincidence that our meticulous care in defining tuning systems happens to correspond visually (meaning geometrically) with a more casual description of the ancients that already spoke of the "octave" 2:1 as if it divided equally into twelve semitones like the hours of the day and of the night although they also understood perfectly well that their own correlations were only approximate. *Plumbline* and *balance beam* were idealized notions borrowed from carpentry and commerce, and their 12-tone octaves were as variable as lunar months, but they drew circles and divided them equally

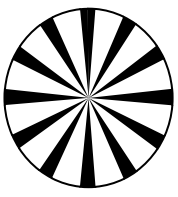
long before they learned to make *tone* numbers approach that geometrical accuracy *proportionally,* for "music" in a general sense became understood as *ratio theory.* The most important single idea seems to have emerged in Mesopotamia with eventual discovery that the difference between the longest day and the shortest night in the latitude of Babylon approximates the difference of a wholetone between a musical fourth and a musical fifth. This assumption links music to the calendar as a cycle of twelve—quite indifferent to each other but not to us, and so tonal cosmology emerges from the mists of history as an almost *natural* human convenience. It is a cultural consequence of our effort to make sense of things, whatever the cost. Discovery that the musical fifth emerges as *two-thirds* of the reference string length (true also on blown pipes under certain conditions to be explained in an essay on pitchpipes) eventually produces the Sumerian deity ENKI (Babylonian EA) as "god 40" (meaning two-thirds of sixty)—the only member of the pantheon wise enough to anticipate the future, and god of the arts and crafts. As a linear time measure we can locate two-thirds of 60 as 40 minutes on the local clock, but as a proportional measure of pitch in the 2:1 musical octave two-thirds correlates with 7 hours in a cycle of 12, and this contrast between two kinds of equality proves endlessly fascinating. From this initial insight we can map the 12 normative tones in a "tone-circle" with as much accuracy as we please. If the first musical fifth of ratio 2:3 embraces seven semitones (or three wholetones and a semitone as Philolaus affirms in the 5th c. BC) and lies at 7:00 o'clock, then all subsequent fifths can be located at 7-hour intervals and enjoy Mesopotamian *mantles of radiance,* a beautiful metaphor from *Enuma Elish,* the Babylonian creation epic dating to the time of Hammurabi (c. 1800 BC). The locus of F C G D A E B and their relatives can be understood without doing any arithmetic. Our oldest tuning texts (for the 9-string Akkadian lyre) date to this period and cover the entire set of thirteen pitch classes in the spiral of fifths, shown here in modern alphabetical notation.

AΦ EΦ BΦ F C G D A E B FΣ CΣ GΣ

We attune ourselves to the cultural foundations of civilization by thinking of creation as proceding symmetrically from the middle by the first ratio (2:3) that creates new pitch classes, mapping this spiral in opposite directions at intervals of seven hours on a local clock. These basic concepts employed in my studies of ancient musical allegory are designed to pry us lose from contemporary habits without losing the advantages of modern precision. But glance at the *cents values* in the harmonic series and notice that the musical fifth is given a "rounded" value of 702 logarithmic *cents.* After 12 such intervals we accumulate a cyclic excess of 24 *cents.* Or if we tune fourths of 3:4 worth only 498 cents we are deficient by the same amount. But every tone is surrounded by a Platonic "no man's land" within which the ear cannot make decisions. And that human disability, which varies widely among us, opens the door to alternative systems. It is a both a blessing and a curse--one of the most powerful metaphors in the Bible. Here lies both *sin* and *salvation.*



Now look back at Figure 1 and notice that two musical thirds of 5:6 + 6:7 compose a musical fifth of 316+267 = 583 logarithmic cents in modern values. Then add together the cents values of the three wholetones defined by 7:8 + 8:9 + 9:10 that compose a logical fourth of 7:10 as a *tritone* of 231+204+182 = 617 cents, now 34 cents *larger* than the tritone fifth of 5:7 worth only 583 cents. Our *system* degenerates into verbal nonsense inviting maximum misunderstanding. And what is the practical result?

The logical ambiguity right here has turned out to be one of the richest resources in musical harmony. *Because* the ear and mind can be surprised and confused composers possess a kind of fulcrum on which to pry us loose from one perspective while a new one dawns on consciousness. A defect from one point of view is a blessing in disguise from another, and in this case it becomes an active ingredient in modern harmony.

But how did the ancients regard this complexity in "Pythagorean" theory? We happen to inherit two remarkable kinds of evidence. Historically first, the Akkadian lyre tunings of Kilmer, Crocker and Brown, dating to c.1800-1600 BC, seize on this ambiguous *tritone* fourth/fifth as the defining element in their version of our seven "white key" *diatonic* modes, for in each modal octave this interval occurs between different pairs of numbered strings. So this earliest image of the tonal cosmos is ultimately defined by ear (and the god Marduk/Baal is born with "four enormous ears"). The world *out there* is understood by analogy with an *inner experience.*

A strictly logical Greek description by Aristotle is a second valuable heritage. He points out that the ear cannot distinguish between a "third-tone" and a "quarter-tone" as defined in fourth century BC Greece (Archytas ratios of 27:28 and 35:36 are excellent approximations). And in the standard wholetone of 8:9 worth 204 cents a twelfth part is 17 cents, exactly half of the 34 cent contradiction we are finding here. Aristotle's figure estimates the general human limits of *excess and deficiency* in *aurally* confirming anything. From his figures we can deduce that among 12 such pitch classes in an octave of six wholetones nearly half of the potential "tone-space" consists of a Platonic "no man's land" surrounding each tone. Modern temperament now equalizes tritone fourths and fifths but their ambiguity remains, and Aristotle's estimate of aural tolerances remains valid.

In this *sea of possibilities* within an *octave* 2:1 digitalized in various ways I welcome Alfred North Whitehead's advice: "The guiding motto in the life of every natural philosopher should be, Seek simplicity and distrust it" (*Concept of Nature,* Ch. 7). It is a personal credo pervading all of these essays.

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