# Addendum: Ancient Harmonics and the Book of Jonah

by Ernest McClain with Duane L. Christensen

The concluding sentence in Jonah 4:11 may be the most brilliant in the Bible, and for several reasons.

> "And as for me, should I not have compassion on Nineveh, that great city which has in it more than 120,000 persons — who cannot discern between their right hand and their left, and much livestock?" (DLC translation).

For the mathematically innocent the layers of meaning prove so fiendishly subtle as to defy any understanding, but the fantasy has been too pleasurable to be spoiled by this brief and apparent *non sequitur* at the end. The number 120,000, however, proves to encompass the whole reach of "Davidic harmonics," and its arithmetical analysis is straight forward:

1) build the matrix;
2) then notice the patterns it contains (familiar only in musicology);
3) rotate the matrix on itself to discover what tonal meanings survive arithmetical reciprocation (when meanings like right and left or up and down reverse—as often happens in the course of time.
4) Whatever remains *invariant* (as members of both *sets*) is the "harmonic" model for "Jewish wisdom."

Nobody needs to know this "musical" arithmetic except for the personal satisfaction of sharing an authorial insight. Generalized meaning is always made clear by other examples in common language that are too blunt to be misunderstood except perversely. Modern computer graphics, however, make it far easier to display authorial models that had to be explained verbally.

We met Jonah earlier as a passenger on a "foreign" ship shown here as the smallest "octave matrix" within *either* YHWH's limit of 48 or the Mesopotamian base 60 normative limit of 60. A set of 7 + 1 (= 8) "Youthful" odd integers turn into "mature fighting men" (most of them successfully "married" by female doubling) and with a biblical life expectancy of 2 x 60 = 120 years. (We are looking here at their "small" octave.) Only five survive "the reversals of fortune." Moses as "the meekest of men" is symbolically 5 and matures into 40 via three doublings ($2^3$ = 8), but becomes a great "leader" only after another doubling into 2 x 40 = 80 in the "great" octave. Moses is the great Jewish equivalent of Plato's "harmonic mean" (i.e. he is reducible to "two-thirds" as in 2/3 x 60 = 40 and 2/3 x 120 = 80) and can well boast that he dies with his "powers" undiminished. "His eye was not dim not his natural force abated" (Deut. 34:7). It was entirely appropriate that "the days of weeping and mourning for Moses" were *thirty,* and that he was permitted to view *the Promised Land* only *from across the river.*

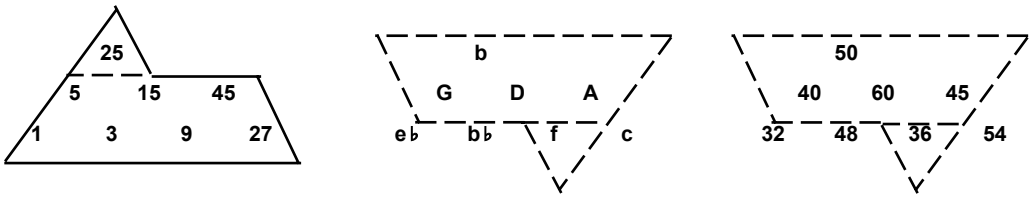


**Figure 1. Jonah's ultimate literary inspiration is "god 50" in the *base 60* octave double 2:1 = 60:30 of ancient Sumer and its descendants, mythologized in the Gilgamesh epic and compressed by Plato to "4:3 mated with the 5." (To be developed elsewhere.) Here "god 50" (Ellil/Enlil) is shown "plowing the earth" as "lord of the plow" (his symbol is the spade). But he is not very bright; he dramatizes the future but without being able to anticipate it. Humor has been a constant friend in the history of harmonic theory.**

We can explore the potentiality of Nineveh by extending this pattern to 120,000, and then doubling all "male, fighting men" to the limit within its half at 60,000. The ascent along the left is by quintupling (double each integer twice in succession and add to itself to determine the next). Progress to the right is by tripling (double each integer in succession and add to itself). For many purposes in studying tonal consequences no further arithmetic is required. Rotation of the matrix exposes the tonal meanings that survive arithmetical reciprocation. This initial multiplication was achieved by "Egyptian duplatio" of very great antiquity, and it remains an easy tool for exhausting tonal patterns within a given numerical limit. Reciprocation, however, is symmetric around 4 x 15 = 60 as pitch class D and constant reference, and base 60 provides a new set of "twin" values that are suppressed here. David musicology, instead, merely extends the *same* matrix arithmetic as far as necessary and accepts the consequences of granting every number double meanings as "multiples and submultiples."

The matrix arithmetic for 120,000 is analyzed first as the multiplication table for 3x5 that produces male, "fighting men." To that limit there are exactly 48. But why this limit matters is the last thing we learn. YHWH's values as 10.5.6.5. sum to 26, and 120,000 is $2^6$= 64 x 1875. His "great" power (*dynamis*) is exponential power.

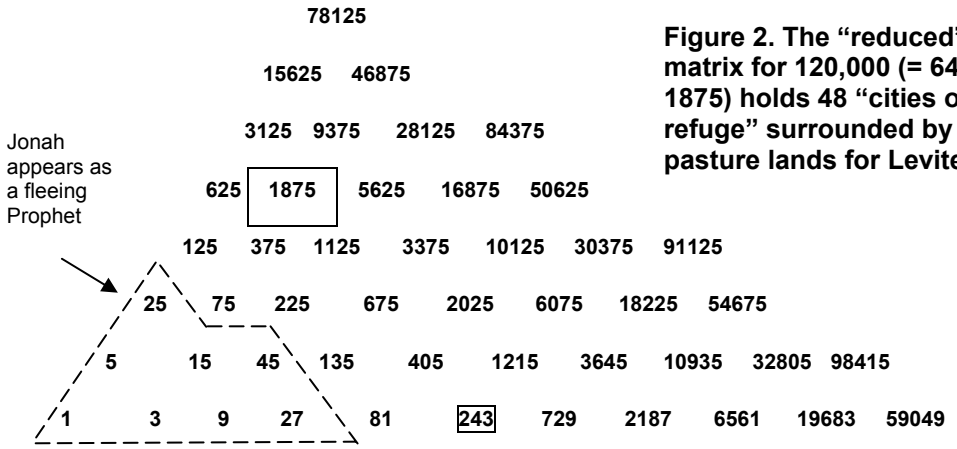


**Figure 2. The "reduced" matrix for 120,000 (= 64 x 1875) holds 48 "cities of refuge" surrounded by pasture lands for Levites.**

**This reduction exposes Abram (whose Hebrew letters sum to 243) as first "ambidextrous" explorer of the Holy Land. These eleven paired symmetries in the bottom row are the maximum possible in any 12-tone tuning system limited to rational numbers.**

The Mesopotamian Ellil (Sumerian Enlil) is basically friendly to mankind (*fivers* like himself) but he is depicted as too rattle-brained to recognize his own "children." He is deficient in the *rationality*—reasoning power—that distinguishes men from animals. *Bible authors are liberating "25 ≈ 50" as a "bird" to "swim" in the skies (i.e., the* "waters above") *after three days in the belly of a great fish* (in the "waters below"). Ancient Sumerian metaphors are being developed with equal authorial astuteness and "tribal" pride. New Testament authors will turn "Fish City" into a Christian heaven where twelve "fishers" of men will "save" a dozen Jonahs.

But notice the basic *irrationality* of the *musical* results. "Jonah" as a *minor third* of 5:6 above and/or below the reference D, defines the interval from f up to b as a musical "tritone" of 3 wholetones (f-g-A-b) [orthography indicates slightly different ratios] but its inverse from b up to f is a *diminished fifth* (b-c-d-e flat-f) [with or without the flat] of 2 wholetones and 2 semitones, and *none* of which can be set accurately by human ears except *indirectly* via the matrix procedure (where only *neighboring matrix tones* can be tuned from each other). The accuracy of the arithmetic thus proves entirely spurious to the ear in these particular circumstances! Octave doubling to the maximum within 2:1 = 120,000:60,000 can be computed *bustrophedon* (from

 

right to left by the ancient) Chinese algorithm "add or subtract one-third." (But the 11th value in row 1 should be doubled first, as shown below.).

**The bracketed four values here are the *a priori* matrix minimum for computing a "YHWH" unequal temperament by 63/50 (=1.26). (Some products must be halved to remain within the octave 60,000:120,000.)**

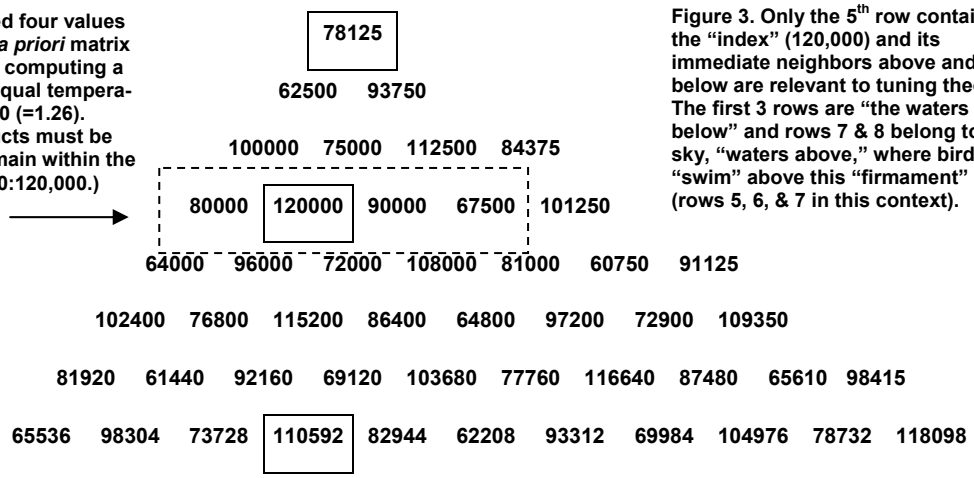


**Figure 3. Only the 5th row containing the "index" (120,000) and its immediate neighbors above and below are relevant to tuning theory. The first 3 rows are "the waters below" and rows 7 & 8 belong to the sky, "waters above," where birds "swim" above this "firmament" (rows 5, 6, & 7 in this context).**

1) In row 3 the "head" digits of the first seven numbers (1024 to 729) "seal" the seven "white-key" heptatonic scales in smallest integers for their characteristic *modal* octaves (on F C G D A E B, in that order). Here they are inflated by common factors of 100 (4 x 25, Jonah's initial value as "dove.")
2) In row 4 the "head" digits of the first five numbers (64 to 81) "seal" the five pentatonic modal patterns as "leaders of thousands," and 108 is the favored Buddhist and Chinese pentatonic seal.
3) In row 5 the first four values display David's minimal "ten thousands" required for temperament because of restrictions in the fourth value (67,500) that prove determinate. The integers 8 and 9 are the harmonic and arithmetic means in the basic octave of 12:6 encoded in the Hebrew letters of David's name as 4.6.4.
4) The peak offers an improved square root of 2 in relation to the 4th counter in the base, and reduces the Marduk matrix to only 8 rows out of its original "flood" of 15, flaunting YHWH's parsimony. 110,592/78,125 is a slightly excessive Marduk square root of 2 as 1.4155; and is admittedly very slightly better than YHWH's 17/12 = 1.4166; but it is too grotesque for the ancients ever to employ in calculation. *Jewish concern is not with "power" but with "wisdom,"* displayed here in mocking Marduk with a more practical "Diophantine approximation." But another lesson lies ahead.

Table 1. Calculating YHWH "Unequal" Temperament.

Multiply "four good men" twice in succession by 63/50 = 1.26 on a modern calculator and halve where necessary to allow the 12th and last tone at 107,163 to remain largest. Realign in numerical scale order and assign any convenient chromatic tone names. "Fish City" provides the appropriate "octave" numerical context. Deviation from modern 12-tone theory is shown in hundredths of a semitone, all well within the modern tolerance of one-tenth of a semitone. As "temperament leader" 675 symbolizes Jacob's Egyptian grandson Ephraim as legendary "musicological bull," for no values need be larger than his. But the ancient calculation had to be carried out in "Egyptian arithmetic" as shown elsewhere, carefully mythologized in Joseph's story. Few tunings can approach this perfection, and none need to do so. Biblical concern is with *Wisdom,* not *Science,* and not out of ignorance, but of competence.

| | | | |
|---|---|---|---|
| 80,000 ± 2 | 60,000 ± 0 | 90,000 ± 2 | 67,500 ± 4 |
| 100,800 ± 2 | 75,600 ± 0 | 56,700 ± 2 | 85,050 ± 4 |
| 63,504 ± 2 | 95,256 ± 0 | 71,442 ± 2 | 107,163 ± 4 |

 

Explanatory Note to Table 1

There is a cumulative cyclic error of 2 cents with each perfect fifth of 3:2, so that within four consecutive "men" there are three such intervals and thus an error of 6 cents (3x2). The reference value 120,000, however, coincides with its half at 60,000 and is second or third among each set of four (depending on which way we count—left to right or right to left) hence the largest error in one direction is 2 cents and in the other is 4 cents. There is no significant error in 1.26 as cube root of 2, for it is never used more than twice in succession in computing these twelve tone numbers. Thus there is no significant error in this YHWH "slightly unequal" temperament so far as the human ear is concerned. YHWH's temperament displays his "personal" rigor (and justice) while his "Just" Jonah octave 48:24 displays his generosity or "forgiveness." In the book of Jonah, justice and mercy are opposite sides of the same coin.

Any "common language" for humankind disappears under this confusion. It is not YHWH who names "Babylon" for *babble* but the Mesopotamian Marduk scribes themselves, amused by the verbal contradictions in their own careful counting. Every diatonic (7-tone, white-key) mode suffers this tritone somewhere within, and ancient Akkadian lyre tunings defined seven modes by the *locus of the tritone,* the *least* reliable of ratios! Our excessively "self-contradictory" prophet is a very ancient musical inheritance. Now we see him, now we don't, like a "joker" in a deck of cards, capable of trumping anyone's ace. Jonah's name, worth 71 in Hebrew, lies outside anybody's tuning theory, inviting playful use as 7 and 1, reversed in 17 as either "head" or "tail," and present even in the notion of an "octave" as *heptatonic.* This doctrine of names as revealing the essence of character is true of YHWH, Adam and Eve, Abram, Sara, and Lot, Moses, Aaron, David, Jesus and an uncertain number of other leading Bible characters now being studied with new attention to *gematria* as *Jewish algebra.* What follows here is Jewish "geometrical algebra" behind the verbal play.

The universe rotates day and night, and Nineveh with it, and so—as the matrix defined by 120,000 -- it rotates on the second counter in its fifth row to illustrate how Jonah, who originally appears as $5^2 = 25$ (first counter in the third row) and so is "left in the shade" in the morning, is also lifted out of the city to fly free in the heavens until he is needed again below. Here I use the inverted Nineveh matrix as the "bush" which grows up in a night to shade our weary prophet. (The upright version could have been used also as the "sea" in which he nearly drowned, but I chose to display the YHWH matrix instead.)

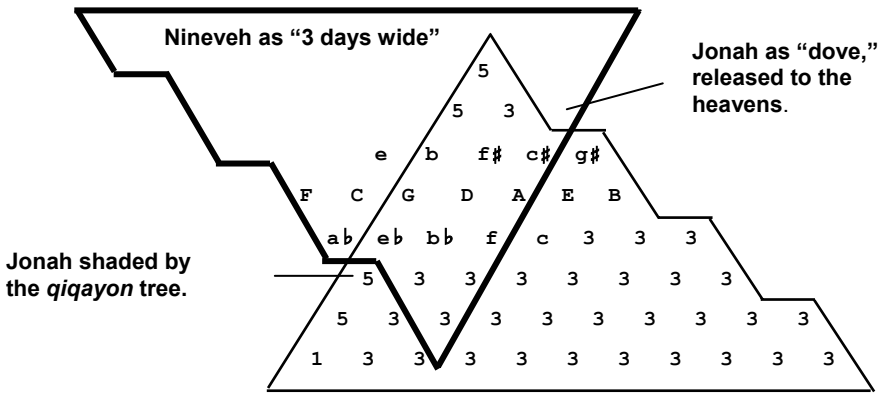


**Figure 4: The Nineveh matrix for 120,000, rotated as Jonah's *qiqayon* shade tree.**

 

The only tones that remain invariant in pentatonic, heptatonic, and 12-tone tunings in either Spiral Fifths, Just, or YHWH Temperament are the first four along the base (1-3-9-27) that sum to 40 "days or nights." Nineveh is "three days across" probably because its construction offers only 3 tones in the fifth row (G D A) that remain invariant under all conditions. They are Plato's "three means" with "D" always geometric mean in any construction while G and A exchange places under reciprocation as Platonic twins (the arithmetic and harmonic means). These three tones are defined by David's name, and in a tone-circle at any hour they always point to the tritone 17/12 across from Deity.

Notice that Nineveh here is a "pentatonic city" whether "Upright" (*Jeshurun*) and masculine or inverted and female, and that alternate 5-tone modes produce the 7-tone "Menorah mode" of the Torah. (The three tones in lower case print *exchange* places under reciprocation.) A slightly uneven Jewish temperament can be computed from either C G D A or from E A D G by YHWH's personal cube root value of 1.26, as shown in Table 1 above. The inflation by "ten thousands" is required to permit this calculation to be used twice on the fourth value in the set (67,500) with an integer result (multiply it by 1.26 squared on a pocket calculator) of 107,163. This twelfth tone of the "Davidic Temperament" thus motivates and determines the entire construction with its many layers of meaning.

The book of *JONAH*, I suggest, must be considered a Jewish candidate for the most profoundly clever mathematical allegory ever conceived in the human mind.

For further information, see:

McClain, Ernest. *The Myth of Invariance: The Origin of the Gods, Mathematics and Music from the Rg Veda to Plato* (Nicolas Hays, Ltd., 1976).

_____. *The Pythagorean Plato: Prelude to the Song Itself* (Nicolas Hays, Ltd., 1978).

_____. *Meditations through the Quran: Tonal Images in an Oral Culture* (Nicolas Hays, Ltd., 1981).

_____. "Musical Theory and Ancient Cosmology," in *The World and I* (Washington, D.C., 1994), pp. 371–391.

_____. "The 'Star of David' as Jewish Harmonical Metaphor," *International Journal of Musicology* 6 (1997), pp. 25–49.

_____. "The Forgotten Harmonical Science of the Bible," *Epigraphical Society Occasional Papers* (forthcoming). A pre-publication version can be downloaded now on www.ernestmcclain.net.

 